

Self-locking of 1-DOF Rigid Origami at a Target Surface

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Abstract

We describe progress in developing solutions of an inverse problem in the design of rigid origami: broadly, we are looking for ways to develop a one degree-of-freedom (1-DOF) rigid origami that self-locks at some desired shape. More formally, given a C^2 , closed, 0-connected and orientable 2-manifold S embedded in \mathbb{R}^3 , for any positive real number $\varepsilon > 0$, find a paper P , which is a connected simple straight-line graph G embedded on a closed 0-connected planar region $A \subset \mathbb{R}^2$, such that

1. G is the set of crease points, and has no cut vertices or bridges.
2. P is rigid-foldable to its final rigidly folded state P' , where some folding angles reach $\pm\pi$ and the folding motion halts.
3. P has one degree of freedom.
4. the Hausdorff distance d between S and P' satisfies $d \leq \varepsilon$.

Callens and Zadpoor (2017) have reviewed inverse problems in origami design. However, if we restrict ourselves to systems that have only one degree of freedom, there are only limited results (Dudte et al., 2016). Here, using a family of quadrilateral crease patterns that are rigid-foldable but not necessarily flat-foldable, we are able to give solutions for a self-locking approximation of a developable surface. An example is shown in Figure 1. Noting that even if all inner vertices are flat-foldable, a 1-DOF rigid origami can approximate a surface that is not necessarily developable (Song et al., 2017). We are working on extending our results to a general surface.

Furthermore, this approximation problem naturally induces an optimization problem, that is, find the “best” paper that fits the extra presupposed requirement, such as minimizing the number of inner vertices, the total bending energy (Solomon et al., 2012); or restricting the minimum length of all the creases, the minimum of sector angles, etc. Some results here will also be discussed.

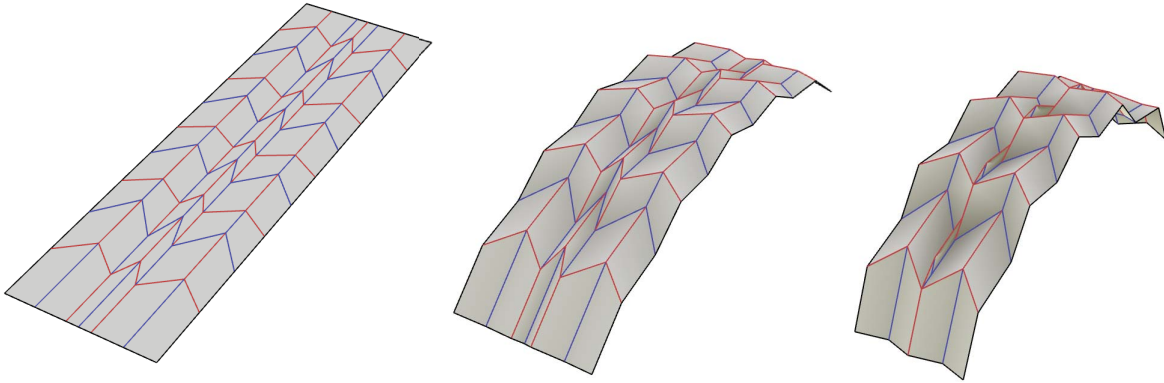


Figure 1: A self-locking approximation for a developable surface in the form of a one-dimensional catenary. The figure shows the folding motion from the initial state to the final rigidly folded state that forms a “rigid origami bridge”. This picture is plotted by Freeform Origami ([Tachi, 2010](#)).

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